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DISCUSSION OF TRUSS DEFLECTIONS BY THE COORDINATE METHOD

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STRUCTURAL DIVISION

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DISCUSSION

L. C. MAUGH,[2] M. ASCE.—An interesting and accurate solution for the common problem of determining the movement of the panel points of triangular trusses is presented in this paper. The method provides an algebraic procedure that corresponds to the graphical solution obtained by use of the Williot-Mohr displacement diagrams. In fact the calculations involve the determination of the coordinates of the various points in the Williot and Mohr diagrams by expressing the components parallel and perpendicular to the members in equation form. By means of such equations the successive increments of motion that any point undergoes because of the change in length of the various members can be determined. Undoubtedly many engineers will prefer such an algebraic procedure to the graphical solution, although it is doubtful if any person can fully understand the algebraic method that is presented unless he is able to use the Williot-Mohr diagrams. Mr. Chu has shown that, once the sign convention is familiar, the calculations can be made without much difficulty.

In many practical problems it is not necessary to determine both coordinates for every joint but only the vertical displacements of some points and the horizontal movement of others. When this condition exists, the writer prefers the application of the virtual work theorem in its most general form. As applied to the calculation of truss displacements this theorem can be stated as follows:

Theorem.—

When the displacements of any two points of a truss are known the movement of a third point can be obtained by applying the virtual work principle to the simplest stable assemblage of members connecting the three points.

This generalized concept of virtual work was developed in detail in a dissertation[3] by M. Megahid in 1948. The application of the principle of virtual work to the problem that Mr. Chu has solved by analytic geometry will be illustrated by a few numerical examples.

If the vertical displacement of point b in Fig. 7 is known, then the horizontal movement of point i can be calculated by considering the virtual forces shown in Fig. 12(a). By the principle of virtual work the work done by the external virtual forces acting through the real displacements must be equal to the work that is obtained from the internal virtual forces acting through the actual change in length of the members. For the forces shown in Fig. 12(a) and the change of lengths given in Fig. 7 this equality of external and internal work gives $1 \times i_x + 2.33\, b_y = -1.0 \times -42.0 + 1.414 \times 59.4 + -1.67 \times$

NOTE.—This paper by Kuang-Han Chu was published in January, 1951, as *Proceedings-Separate No. 54*. The numbering of footnotes, tables, equations, and illustrations in this Separate is a continuation of the consecutive numbering used in the original paper.

[2] Prof., Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

[3] "The Free Main System," by M. Megahid, dissertation presented to Fouad I University, Giza, Egypt, in 1948, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

+ 40. If the value of b_y is 79.6 downward, then the value of i_x is $i_x + 2.33 \times 79.6 = 59.3$; and $i_x = -126.4$. The minus sign indicates that the displacement is opposite in direction to the assumed force.

The horizontal displacement of point b can now be determined by means of the virtual forces in Fig. 12(*b*). Again the external work of the virtual forces in Fig. 12(*b*) acting through the actual displacements in Fig. 7 are equated to the corresponding internal work; thus: $1 \times b_x - 0.571 \times 126.4 = -0.571 \times -42.0 + 0.715 \times +40.0 + 0.810 \times +59.4$, from which $b_x = 172.8$.

The vertical displacement of point c is easily determined by means of the virtual forces shown in either Fig. 12(*c*) or Fig. 12(*d*). If the force system in Fig. 12(*c*) is used, the following equation is obtained: $1 \times C_y - 2 \times 79.6 + 1 \times 42.0 = 1.12 \times -47.0 + -1.12 \times +73.8 + -0.5 \times -43.0 + -1.414 \times +59.4 + 1.0 \times 42.0$, from which $C_y = -122.7$ (upward). If the force system in Fig. 12(*d*) is used, the equation becomes: $1 \times C_y - 1 \times 79.6 + 1 \times 172.8 - 1 \times 126.4 - 42.0 = 1.12 \times -47.0 + -1.12 \times +73.8 + -0.5 \times -43.0$, which gives $C_y = -122.7$.

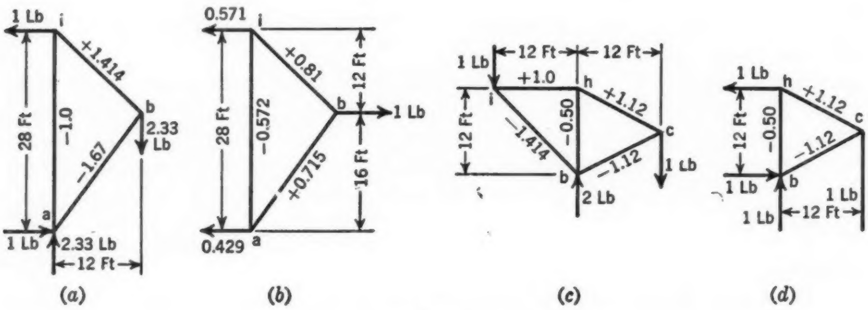

FIG. 12.

The procedure that is used in the foregoing examples has the advantage of involving only a few members of the truss and only one sign convention. It is interesting to note that for every geometrical solution there is an analogous solution by the principle of virtual work.

KUANG-HAN CHU,[4] JUN. ASCE.—The writer appreciates the comments made by Mr. Maugh and his introduction of Mr. Megahid's method. This method can be used advantageously for problems in which, as Mr. Maugh states, it is not necessary to determine both coordinates for every joint of a truss but only for the vertical displacements of some points and the horizontal movement of others. The method involves a few members of the truss and only one sign convention. However, Mr. Megahid's method becomes tedious when the displacements of many joints are needed, as it is necessary to find a new stress pattern for each displacement of each joint although only a few members are involved. The Williot and Mohr method has been proved useful for many

[4] Structural Designer, D. B. Steinman, New York, N. Y.

practical problems, and it is discussed in detail by Mr. Maugh in his book[5] because "* * * it provides an extremely practical method for the determination of the actual movement of all joints." The writer's method inherits the advantages and disadvantages of the Williot and Mohr method, except that the writer's method is an algebraic one and therefore, can be carried to any desired degree of accuracy.

After discussions had been formally closed, the writer's attention was called to a discussion submitted by Mr. Yen-chi Lu of the National Taiwan University (Formosa, China). Since it was received too late to be processed as a formal discussion, two main points brought out in this discussion that are worth noting are presented as follows:

1. Mr. Lu recommends that the magnitude and the signs of the components be determined by the following equations:

$$(\Delta_{QP})_x = \delta_{QP} \cos \phi_{QP} \quad \text{(18a)}$$

$$(\Delta_{QP})_y = \delta_{QP} \sin \phi_{QP} \quad \text{(18b)}$$

in which ϕ_{QP} is the angle that the bar QP makes with the x-axis through the starting point Q in conventional orientation—counterclockwise as positive (see Fig. 13). The main advantage of using Eqs. 18, as Mr. Lu states, is that the signs of Δ components are automatically determined and the four rules as outlined by the writer are unnecessary. However, in actual applications, the writer's method may be more expedient than Mr. Lu's because in using Eqs. 18 one has to watch carefully the signs of δ, cos ϕ, and sin ϕ, and then carry out the multiplication to determine the signs of Δ components. In the writer's method, the signs can be determined by merely following the rules as shown in Fig. 2. Eqs. 18, however, are easy to remember and express the mathematical meaning of the writer's rules more clearly and concisely.

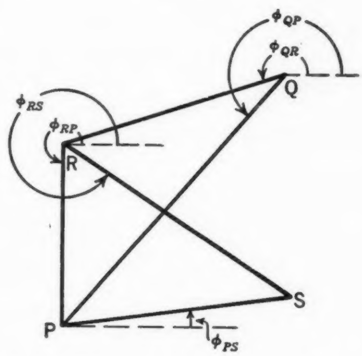

FIG. 13.—DIAGRAM SHOWING ANGLE $+\phi$ WITH Q, R, AND P IN TURN AS STARTING POINTS

2. In case the actual displacements of the point M of known condition of motion, $\overrightarrow{F''M'}_x$ and $\overrightarrow{F''M'}_y$, are known, Mr. Lu recommends the use of the following equations to determine the values of x''_{FM} and y''_{FM}.

$$x''_{FM} = x'_{FM} - \overrightarrow{F''M'}_x \quad \text{(19a)}$$

and

$$y''_{FM} = y'_{FM} - \overrightarrow{F''M'}_y \quad \text{(19b)}$$

[5] "Statically Indeterminate Structures," by L. C. Maugh, John Wiley & Sons, Inc., New York, N. Y., 1946, p. 40.

Eqs. 19 can be derived from the writer's Eqs. 12. In the example given by the writer, $\overrightarrow{F''M'}_x = \overrightarrow{F'M'''}_x$ and $\overrightarrow{F''M'}_y = \overrightarrow{F'M'''}_y$. Therefore, the coordinates of the points c''_M and d''_M can be determined as shown in Table 4. Thus, items 8 to 22 in Table 2 can be replaced by the six items as shown in Table 4, which is a material simplification.

TABLE 4.—COMPUTATION OF COORDINATES

Item[a]	Joint	x	Computation	y
(a) Point c''_M				
7	c'''_M	[+154.28]		[+122.38]
9	c'	+55.04		+230.64
14	c''_M	[−99.24]	(Item 9) − (Item 7)	[+108.26]
(b) Point d''_M				
7	d'''_M	+154.28		+122.38
16	d'	−33.84		−128.44
20	d''_M	[−188.12]	(Item 16) − (Item 7)	[−250.82]

[a] Item numbers refer to Table 2.

Besides the two points presented, Mr. Lu also proposes a table form and gives equations involving trigonometric functions of ϕ based on the same principles as outlined by the writer. They are not presented here because of the following considerations:

a. Mr. Lu's equations are essentially the same as the writer's but more complicated in form; and

b. Different engineers can make different tabular forms for their own convenience.

In conclusion, the writer wishes to thank Mr. Lu for his contributions.

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* Deceased.